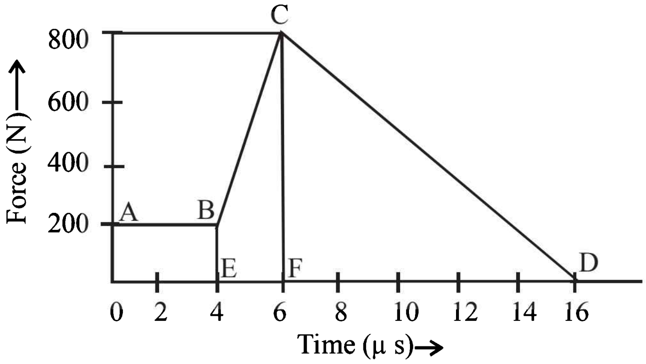
CHAPTER 5 Momentum and Impulse

Section-A JEE Advanced/IIT-JEE

A Fill in the Blanks

1. A particle of mass 4 m which is at rest explodes into three fragments. Two of the fragments each of mass *m* are found to move with a speed *v* each in mutually perpendicular directions. The total energy released in the process of explosion is .... *(1987 - 2 Marks)*

2. The magnitude of the force (in newtons) acting on a body varies with time *t* (in micro seconds) as shown in the fig *AB*, *BC* and *CD* are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4$ μs to $t = 16$μs is ....Ns. *(1994 - 2 Marks)*

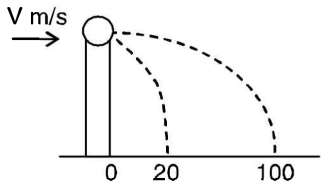


C MCQs with One Correct Answer

1. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_o. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_o$ while still moving in air. The value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is *(2001S)*
   (a) zero (b) $(m_1 + m_2)gt_o$
   (c) $\frac{1}{2}(m_1 + m_2)gt_o$ (d) $2(m_1 + m_2)gt_o$

2. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is *(2002S)*
   (a) 30 m/s (b) 20 m/s
   (c) 10 m/s (d) 5 m/s

3. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is *(2011)*



   (a) 250 m/s (b) $250\sqrt{2}$ m/s
   (c) 400 m/s (d) 500 m/s

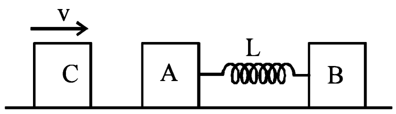
4. A particle of mass *m* is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is *(JEE Adv. 2013)*
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4} + \alpha$ (c) $\frac{\pi}{2} - \alpha$ (d) $\frac{\pi}{2}$

D MCQs with One or More than One Correct

1. A ball hits the floor and rebounds after an inelastic collision. In this case *(1986 - 2 Marks)*
   (a) the momentum of the ball just after the collision is the same as that just before the collision.
   (b) the mechanical energy of the ball remains the same in the collision
   (c) the total momentum of the ball and the earth is conserved
   (d) the total energy of the ball and the earth is conserved

2. A shell is fired from a cannon with a velocity *v* (m/sec.) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec.) of the other piece immediately after the explosion is *(1986 - 2 Marks)*
   (a) $3v \cos\theta$ (b) $2v \cos\theta$
   (c) $\frac{3}{2}v \cos\theta$ (d) $\sqrt{\frac{3}{2}}v \cos\theta$

3. Two blocks A and B, each of mass m, are connected by a massless spring of natural length L and spring constant K. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in fig.. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then ***(1993-2 Marks)***



(a) the kinetic energy of the A-B system, at maximum compression of the spring, is zero.
(b) the kinetic energy of the A-B system, at maximum compression of the spring, is $mv^2/4$.
(c) the maximum compression of the spring is $v\sqrt{(m/K)}$
(d) the maximum compression of the spring is $v\sqrt{(m/2K)}$

4. The balls, having linear momenta $\vec{p}_1 = \vec{p}i$ and $\vec{p}_2 = -\vec{p}i$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2. ***(2008)***

(a) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$
(b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$
(c) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
(d) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

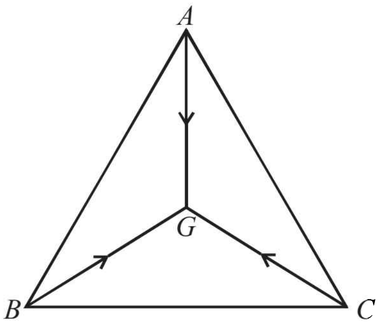
5. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses? ***(2010)***
(a) Total momentum of the system is 3 kg ms^{-1}
(b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
(c) Kinetic energy of the centre of mass is 0.75 J
(d) Total kinetic energy of the system is 4J

6. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision ***(JEE Adv. 2013)***
(a) The speed of the particle when it returns to its equilibrium position is u_0
(b) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$
(c) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$
(d) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

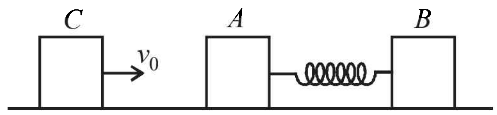
E Subjective Problems

1. A body of mass m moving with velocity V in the X-direction collides with another body of mass M moving in Y-direction with velocity v. They coalesce into one body during collision. Calculate : ***(1978)***
(i) the direction and magnitude of the momentum of the final body.
(ii) the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses.

2. Three particles A, B and C of equal mass move with equal speed V along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed V. What is the velocity of C ? ***(1982 - 2 Marks)***



3. Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig.

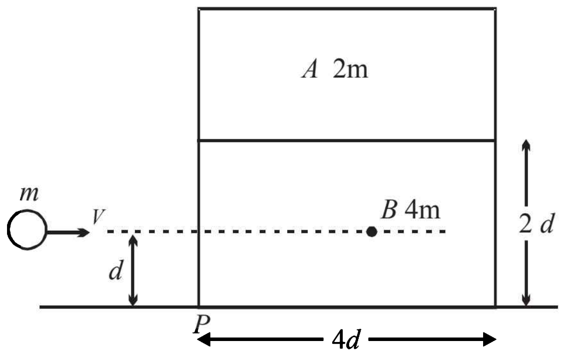


At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of A and B at time t_0; and (ii) the spring constant. ***(1984- 6 Marks)***

4. A ball of mass 100 gm is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **(1985 - 8 Marks)**

5. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3 metres and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find
(i) the angle θ ;
(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. Take $g = 10\ m/s^2$

6. A block '*A*' of mass 2m is placed on another block '*B*' of mass 4m which in turn is placed on a fixed table. The two blocks have a same length 4*d* and they are placed as shown in fig. The coefficient of friction (both static and kinetic) between the block '*B*' and table is μ. There is no friction between the two blocks. A small object of mass *m* moving horizontally along a line passing through the centre of mass (cm.) of the block *B* and perpendicular to its face with a speed *v* collides elastically with the block *B* at a height *d* above the table. ***(1991 - 4 + 4 Marks)***



(a) What is the minimum value of *v* (call it v_0) required to make the block *A* topple ?

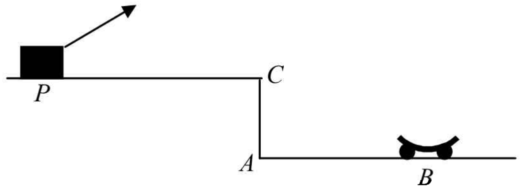
(b) If $v = 2v_0$, find the distance (from the point *P* in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision).

7. A cart is moving along $+x$ direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in *y-z* plane making an angle of 30° with vertical *z*-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length *L*. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine : ***(1997 - 5 Marks)***

(i) The speed of the combined mass immediately after the collision with respect to an observer on the ground,

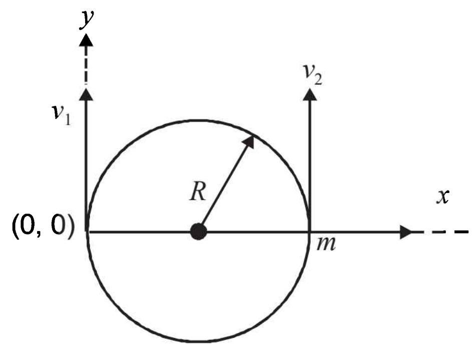
(ii) The length *L* of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

8. A car *P* is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point *B* as shown in figure. The height *AC* is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. ***(2001 - 10 Marks)***



At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

9. A particle of mass *m*, moving in a circular path of radius *R* with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the +ve *y*-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. the man as a function of time. ***(2003 - 2 Marks)***



H Assertion & Reason Type Questions

1. **STATEMENT–1 :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. ***(2007)***
STATEMENT–2 : In an elastic collision, the linear momentum of the system is conserved.
(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.
(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False.
(d) Statement–1 is False, Statement–2 is True.

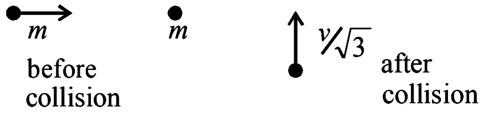
I Integer Value Correct Type

1. A bob of mass *m*, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass *m* suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is ***(JEE Adv. 2013)***

Section-B JEE Main / AIEEE

1. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[2004]**
   (a) Two (b) Four
   (c) One (d) Three

2. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1^{st} mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2^{nd} mass after collision. **[2005]**

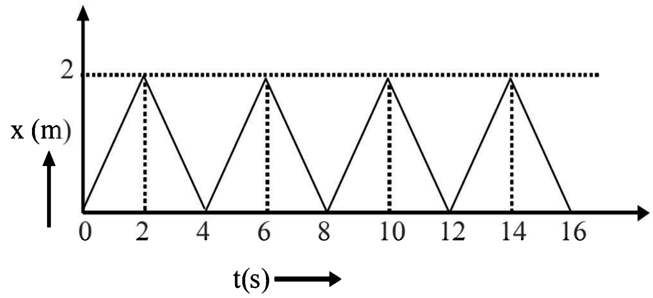


   (a) $\sqrt{3}v$ (b) v
   (c) $\frac{v}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}v$

3. A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velolcity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is **[2006]**
   (a) 144 J (b) 288 J
   (c) 192 J (d) 96 J

4. **Statement -1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
   Statement -2 : Principle of conservation of momentum holds true for all kinds of collisions. **[2010]**
   (a) Statement -1 is true, Statement -2 is true ; Statement -2 is the correct explanation of Statement -1.
   (b) Statement -1 is true, Statement -2 is true; Statement -2 is **not** the correct explanation of Statement -1
   (c) Statement -1 is false, Statement -2 is true.
   (d) Statement -1 is true, Statement -2 is false.

5. The figure shows the position–time ($x - t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[2010]**



   (a) 0.4 Ns (b) 0.8 Ns
   (c) 1.6 Ns (d) 0.2 Ns

6. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[JEE Main 2013]**
   Statement - I: A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.
   Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.
   (a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.
   (b) Statement-I is true, Statment - II is true, Statement - II is not the correct explanation of Statement - II.
   (c) Statement - I is true, Statment - II is false.
   (d) Statement - I is false, Statment - II is true.

7. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed **v**. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[JEE Main 2015]**
   (a) 56% (b) 62%
   (c) 44% (d) 50%

5 Momentum and Impulse

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{3}{2}mv^2$ 2. 0.005 Ns

C 1. (d) 2. (c) 3. (d) 4. (a)

D 1. (c, d) 2. (a) 3. (b, d) 4. (a, d) 5. (a, c) 6. (a, d)

E 1. **(i)** $\theta = \tan^{-1}\left(\frac{Mv}{mV}\right), \sqrt{m^2V^2 + M^2v^2}$; **(ii)** $\frac{mM(v^2+V^2)}{(m+M)(mV^2+Mv^2)}$

2. V, opposite direction to the retraced velocity of B 3. $\frac{v_0}{3}, \frac{2mv_0^2}{3x_0^2}$ 4. 6.53 sec

5. 37°, 122.6 m, 46 m 6. **(a)** $\frac{5}{2}\sqrt{6\mu g d}$ **(b)** $6d\sqrt{3\mu}$

7. 2.5 m/s, 0.318 m 8. 12 s, 15.75 m/s 9. $-mv_2 \sin\omega t\hat{i} + m(v_2\cos\omega t - v_1)\hat{j}$ where $\omega = \frac{v_2}{R}$

H 1. (d)

I 1. 5

Section-B : JEE Main/ AIEEE

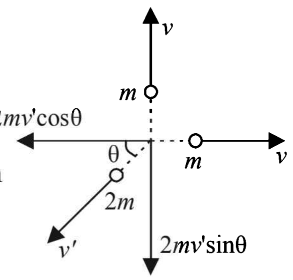
1. (d) 2. (d) 3. (b) 4. (a) 5. (b) 6. (d) 7. (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $2mv'\cos\theta = mv$ ... (i)

$2mv'\sin\theta = mv$



$\Rightarrow \sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

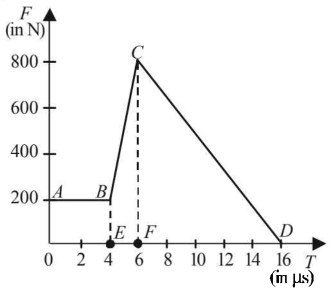
Putting this value in equation (i), we get

$$\frac{2mv'}{\sqrt{2}} = mv, \quad v' = \frac{v}{\sqrt{2}}$$

$$\text{Total } K.E. = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

2. **KEY CONCEPT** Area under the $F - t$ graph gives the impulse imparted to the body.



The magnitude of total impulse of force on the body from $t = 4$ μs to $t = 16$ μs

= area $(BCDFEB)$

= area of $BCFEB$ + area $CDFC$

$$= \frac{1}{2}(200+800)\times 2\times 10^{-6} + \frac{1}{2}\times 10\times 800\times 10^{-6}$$

$= 0.001 + 0.004 = 0.005$ Ns

C. MCQs with ONE Correct Answer

1. **(d)** If we consider the two particles as a system then the external force acting on the system is the gravitational pull $(m_1 + m_2)g$.

$$F_{ext} = \frac{\Delta p}{\Delta t}$$

$$\therefore \Delta p = F_{ext}\,\Delta t = \left(m_1\vec{v}_1' + m_2\vec{v}_2'\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)$$

$$= (m_1 + m_2)\,g \times 2t_0$$

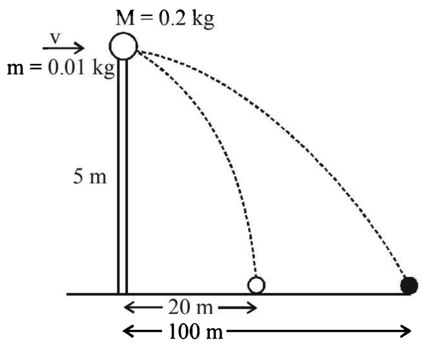
2. **(c)** Just after collision

$$v_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{10\times 14 + 4\times 0}{10+4} = 10\,\text{m/s};$$

Note : Spring force is an internal force, it cannot change the linear momentum of the (two mass + spring) system. Therefore v_c remains the same.

3. **(d)** *For vertical motion of bullet or ball*

$u = 0,\ s = 5\text{m},\ \ t = ?,\ \ a = 10\text{m/s}^2$



$$S = ut + \frac{1}{2}at^2 \Rightarrow 5 = \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow t = 1 \text{ sec}$$

For horizontal motion of ball

$$x_{ball} = V_{ball}\, t \Rightarrow 20 = V_{ball} \times 1 = V_{ball}$$

For horizontal motion of bullet

$$x_{bullet} = V_{bullet} \times t \Rightarrow 100 = V_{bullet} \times 1 = V_{bullet}$$

Applying conservation of linear momentum during collision, w e get

$$mV = mV_{bullet} + MV_{ball}$$

$$0.01\ V = 0.01 \times 100 + 0.2 \times 20$$

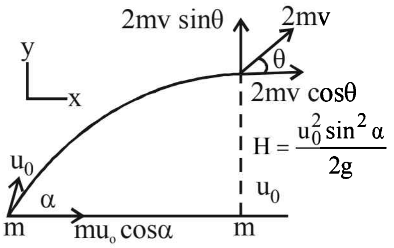
$$\therefore V = \frac{5}{0.01} = 500 \text{ m/s}$$

4. **(a)** Activity B to M for particle thrown upwards

$$v_1^2 - u_0^2 = 2(-g)\left[\frac{u_0^2 \sin^2 \alpha}{2g}\right]$$

$$\therefore v_1^2 = u_0^2(1 - \sin^2 \alpha) = u_0^2 \cos^2 \alpha$$

$$\therefore v_1 = u_0 \cos \alpha \qquad ...(i)$$



Applying conservation of linear momentum in Y-direction

$2mv \sin\theta = mv_1 = mu_0 \cos\alpha$ ...(ii) [from (i)]

Applying conservation of linear momentum in X-direction

$2mv \cos\theta = mu_0 \cos\alpha$ ...(iii)

on dividing (ii) and (iii) we get

$$\tan \theta = 1 \qquad \therefore \theta = \frac{\pi}{4}$$

D. MCQs with ONE or MORE THAN ONE Correct

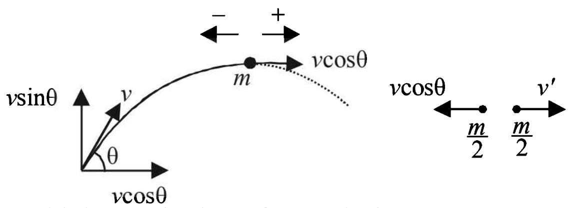
1. **(c,d)** (a) is wrong because the momentum of ball changes in magnitude as well as direction.

(b) is wrong because on collision, some mechanical energy is converted into heat, sound energy.

(c) is correct because for earth + ball system the impact force is an internal force.

(d) is correct.

2. **(a)** As one piece retraces its path, the speed of this piece just after explosion should be $v \cos\theta$



(At highest point just after explosion)

NOTE THIS STEP

Applying conservation of linear momentum at the highest point;

$$m(v\cos\theta) = \frac{m}{2} \times v' - \frac{m}{2} \times v\cos\theta$$

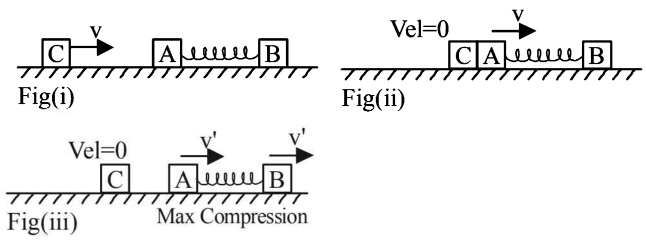
$$3v\cos\theta = v'$$

3. **(b, d)** In situation 1, mass C is moving towards right with velocity v. A and B are at rest.

In situation 2, which is just after the collision of C with A, C stop and A acquires a velocity v. [head-on elastic collision between identical masses]

When A starts moving towards right, the spring suffer a compression due to which B also starts moving towards right. The compression of the spring continues till there is relative velocity between A and B. When this relative velocity becomes zero, both A and B move with the same velocity v' and the spring is in a state of maximum compression.

Applying momentum conservation in situation 1 and 3,



$$mv = mv' + mv' \Rightarrow v' = \frac{v}{2}$$

$\therefore$ K.E. of the system in situation 3 is

$$\frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 = mv'^2 = \frac{mv^2}{4} \qquad \left(\because v' = \frac{v}{2}\right)$$

This is the kinetic energy possessed by $A - B$ system (since, C is at rest).

Let x be the maximum compression of the spring.

Applying energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{4}mv^2 \quad \therefore \quad x = v\sqrt{\frac{m}{2K}}$$

4. **(a, d) KEY CONCEPT**

Use law of conservation of linear momentum.

The initial linear momentum of the system is $p\hat{i} - p\hat{i} = 0$

Therefore the final linear momentum should also be zero.

Option a :

$p_1' + p_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$ = Final momentum.

It is given that a_1, b_1, c_1, a_2, b_2 and c_2 have non-zero values. If $a_1 = x$ and $a_2 = -x$. Also if $b_1 = y$ and $b_2 = -y$ then the $\hat{i}$ and $\hat{j}$ components become zero. But the third term having $\hat{k}$ component is non-zero. This gives a definite final momentum to the system which violates conservation of linear momentum, so this is an incorrect option.

Option d:

$p_1' + p_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0$ because $b_1 \neq 0$

Following the same reasoning as above this option is also ruled out.

5. **(a, c)**

According to law of conservation of linear momentum

$1 \times u_1 + 5 \times 0 = 1(-2) + 5(v_2)$

$\Rightarrow u_1 = -2 + 5v_2$ ...(i)

The coefficient of restituition

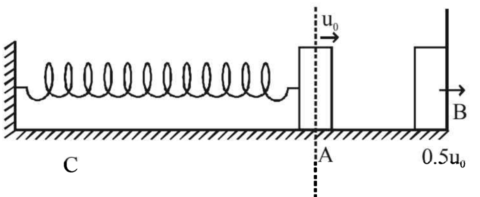
$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - (-2)}{u_1 - 0}$$

$\Rightarrow u_1 = v_2 + 2$ ...(ii)

On solving (i) & (ii) we get desired results.

6. **(a, d)** The particle collides elastically with rigid wall. Here

$$e = \frac{V}{0.5u_0} = 1 \qquad \therefore V = 0.5u_0$$



Equilibrium position

i.e. the particle rebounds with the same speed. Therefore the particle will return to its equilibrium position with speed u_0. option (a) is correct.

The velocity of the particle becomes $0.5u_0$ after time t. Then using the equation $V = V_{max} \cos wt$ we get $0.5u_0 = u_0 \cos wt$

$$\therefore \frac{\pi}{3} = \frac{2\pi}{t} \times T \qquad \therefore t = \frac{T}{6}$$

The time period $T = 2\pi\sqrt{\frac{m}{k}}$. Therefore $t = \frac{\pi}{3}\sqrt{\frac{m}{k}}$

The time taken by the particle to pass through the equilibrium for the first time $= 2t = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$. Therefore option (b) is incorrect

The time taken for the maximum compression

$= t_{AB} + t_{BA} + t_{AC}$

$$= \frac{\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{3}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right]$$

$= \frac{7\pi}{6}\sqrt{\frac{m}{k}}$. Therefore option c is incorrect.

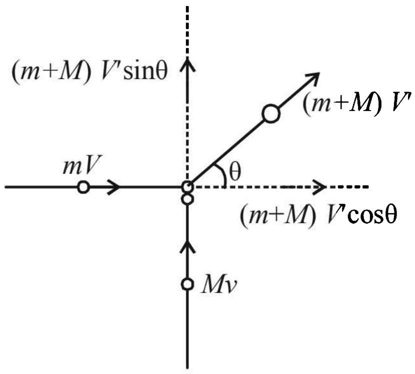
The time taken for particle to pass through the equilibrium position second time

$$= 2\left[\frac{\pi}{3}\sqrt{\frac{m}{k}}\right] + \pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}\left(\frac{2}{3} + 1\right) = \frac{5}{3}\pi\sqrt{\frac{m}{k}}$$

option (d) is correct.

E. Subjective Problems

1. Let V' be the velocity of the final body after collision. Suppose, V' makes an angle θ with x-direction.



(i) Applying conservation of linear momentum in X direction

$(m + M)V'\cos\theta = mV$ ... (i)

Applying conservation of linear momentum in Y direction

$(m + M)V'\sin\theta = Mv$ ... (ii)

Dividing equation (i) and (ii)

$$\tan\theta = \frac{Mv}{mV} \Rightarrow \theta = \tan^{-1}\left(\frac{Mv}{mV}\right)$$

This gives the direction of the momentum of the final body.

Squaring and adding (i) and (ii), we get

$(m + M)^2 V'^2 \cos^2\theta + (m + M)^2 V'^2 \sin^2\theta$
$= m^2V^2 + M^2v^2$

$$\therefore \quad V' = \frac{\sqrt{m^2V^2 + M^2v^2}}{m + M}$$

Thus the magnitude of the momentum of the final body

$= (m + M)V' = \sqrt{m^2V^2 + M^2v^2}$

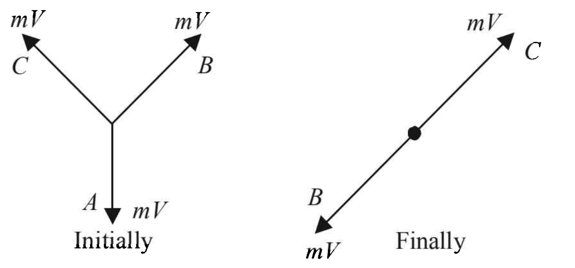
(ii) $$\frac{K.E._i - K.E._f}{K.E._i} = 1 - \frac{K.E._f}{K.E._i} = 1 - \frac{\frac{1}{2}(m + M)V'^2}{\left[\frac{1}{2}mV^2 + \frac{M}{2}v^2\right]}$$

$$\frac{\Delta KE}{K.E._i} = 1 - \frac{(m+M)\dfrac{m^2V^2 + M^2v^2}{(m+M)^2}}{mV^2 + Mv^2}$$

$$\therefore \quad \frac{K.E._i - K.E._f}{K.E._i} = 1 - \frac{m^2V^2 + M^2v^2}{(m+M)(mV^2 + Mv^2)}$$

$$= \frac{m^2V^2 + mM^2v^2 + MmV^2 + M^2v^2 - m^2V^2 - M^2v^2}{(m+M)(mV^2+Mv^2)}$$

$$= \frac{mM(v^2+V^2)}{(m+M)(mV^2+Mv^2)}$$

2. 

Initially By symmetry, the momentum of the system is zero.
Finally The momentum of the system should be zero.
$\therefore$ mV = mV' $\Rightarrow$ V' = V
The velocity of C is V and is in opposite direction to the retraced velocity of B as shown in the figure.

3. Since the collision between C and A is elastic and their masses are equal and A was initially at rest, therefore the result of collision will be that C will come to rest and A will initially start moving with a velocity v_0. But since A is connected to B with a spring, the spring will get compressed.

At $t = t_0$, the velocities of A and B become same.
Applying energy conservation;

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}2mv'^2 + \frac{1}{2}kx_0^2$$

where x_0 is the compression in the spring at $t = t_0$

$$\therefore \quad v_0^2 = 3v'^2 + \frac{k}{m}x_0^2 \qquad \text{... (i)}$$

Applying momentum conservation, we get

$$mv_0 = mv' + 2mv' \quad \therefore \quad v' = \frac{v_0}{3} \qquad \text{... (ii)}$$

From (i) and (ii)

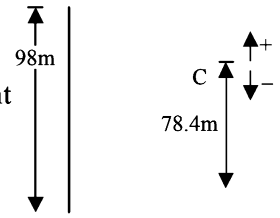
$$v_0^2 - 3\times\frac{v_0^2}{9} = \frac{k}{m}x_0^2 \quad \Rightarrow k = \frac{2mv_0^2}{3x_0^2}$$

4. For the ball thrown up

$$v_1^2 - u_1^2 = 2a_1s_1$$

$$\therefore \quad v_1^2 - 2401 = 19.6h \qquad \text{... (i)}$$

$$s_1 = u_1t_1 + \frac{1}{2}a_1t_1^2$$



$h = 49t - 4.9\,t^2$ ... (ii)

For the ball dropped from height

$$v_2^2 - u_2^2 = 2a_2s_2$$

$$\therefore \quad v_2^2 = 19.6\,(98 - h) \quad \text{... (iii)}$$

$$s_2 = u_2t_2 + \frac{1}{2}a_2t_2^2$$

$98 - h = 4.9\,t^2$ ... (iv)

From (ii) and (iv)

$98 - (49t - 4.9t^2) = 4.9t^2 \quad \therefore \quad 98 - 49t = 0$

$\therefore \quad t = 2$ sec

$\therefore \quad h = 49\times 2 - 4.9\times 2^2 = 78.4$ m (from (ii))

Substituting this value of h in (i) and (ii), we get

$v_1^2 - 2401 = -19.6\times 78.4 \qquad v_2^2 = 19.6\,(98 - 78.4)$

$v_1^2 = 864.36 \qquad \Rightarrow v_2^2 = 384.16$

$v_1 = 29.4$ m/s $\qquad \Rightarrow v_2 = 19.6$ m/s

Note : At point C where the two bodies collide, thereafter both bodies stick and behave as a single body.

Thus, we apply conservation of linear momentum, which gives

$$m_1v_1 - m_2v_2 = 2mv$$

$$\therefore \quad v = \frac{v_1 - v_2}{2} = \frac{29.4 - 19.6}{2} = 4.9\text{ m/s}$$

For the combined body

$u = 4.9$ m/s; $s_1 = -78.4$; $a_1 = -9.8$ m/s^2; $t = ?$

$$s = ut + \frac{1}{2}at^2 \Rightarrow -78.4 = 4.9t - 4.9t^2$$

$$\therefore \quad t^2 - t - 16 = 0 \Rightarrow t = \frac{1\pm\sqrt{1+64}}{2} = 4.53$$

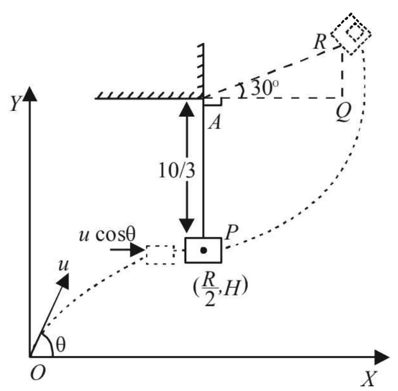
Total time = 4.53 + 2 = 6.53 sec.

5. (i) In ΔAQR

$$\sin 30° = \frac{QR}{10/3},\ QR = \frac{5}{3}$$

$u = 50$ m/s (Given)

At the highest point P, the velocity of the bullet = $u\cos\theta$



Applying conservation of linear momentum at the highest point

$$M(u\cos\theta) + 3M\times 0 = (M+3M)\,v$$

$v = \dfrac{Mu\cos\theta}{4M} = \dfrac{u\cos\theta}{4}$

Applying energy conservation principle for P and R

K.E. of the bullet-mass system at P = P.E. of the bullet-mass system at R

$\dfrac{1}{2}(4M)v^2 = (4M)gh$

$\dfrac{1}{2}(4M)\dfrac{u^2\cos^2\theta}{16} = 4Mg \times \left(\dfrac{10}{3} + \dfrac{5}{3}\right)$

$\cos^2\theta = \dfrac{9.8 \times 5 \times 2 \times 16}{50 \times 50} \Rightarrow \theta = 37°$

(ii) $\dfrac{R}{2} = \dfrac{u^2 \sin 2\theta}{2g} = \dfrac{50 \times 50 \sin 2 \times 37°}{2 \times 9.8} = 122.6\,\text{m}$

$H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{50 \times 50 \times (\sin 37°)^2}{2 \times 9.8} = 46\,\text{m}$

6. **KEY CONCEPT**

Since the collision is elastic in nature applying conservation of linear momentum and conservation of kinetic energy

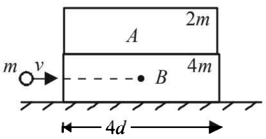
$mv = (4m)\,u + mv'$

where u is the velocity of mass 4m after collision and v' is the velocity of mass $2m$

$\Rightarrow \quad v' = v - 4u$ ... (i)

Also, $\dfrac{1}{2}mv^2 = \dfrac{1}{2}(4m)u^2 + \dfrac{1}{2}mv'^2$

$\Rightarrow \quad v^2 = 4u^2 + v'^2$ ... (ii)



From (i) to (ii)

$v^2 = 4u^2 + (v - 4u)^2 \Rightarrow u = \dfrac{2v}{5}$

Block B starts moving but the block A remains at rest as there is no friction between A and B.

For block A to topple, block B should move a distance $2d$.

Let the retardation produced in B due to friction force between B and the table be a

$F = \mu N \Rightarrow (4\,m)a = \mu\,(6\,mg) \Rightarrow a = 1.5\,\mu g$

For the motion of B,

$u = \dfrac{2v}{5}, v = 0, s = 2\text{d}, a = -1.5\,\mu g$

Now, $v^2 - u^2 = 2as \Rightarrow (0)^2 - \left(\dfrac{2v}{5}\right)^2 = 2(-1.5\mu g)2d$

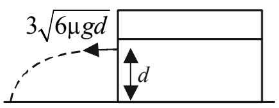
$\Rightarrow \quad v = \dfrac{5}{2}\sqrt{6\mu gd}$

For elastic collision between two bodies

$v_1 = \dfrac{(m_1 - m_2)u_1}{m_1 + m_2} + \dfrac{2m_2u_2}{m_1 + m_2}$

Here $m_1 = m, m_2 = 4m, u_1 = 5\sqrt{6\mu gd}, u_2 = 0$

$\Rightarrow \quad v_1 = \dfrac{(m - 4m)5\sqrt{6\mu gd}}{m + 4m} + 0$



$= -3 \times 5\dfrac{\sqrt{6\mu gd}}{5} = -3\sqrt{6\mu gd}$

Note : The negative sign shows that the mass m rebounds. It then follows a projectile motion.

Considering the vertical direction motion of this projectile.

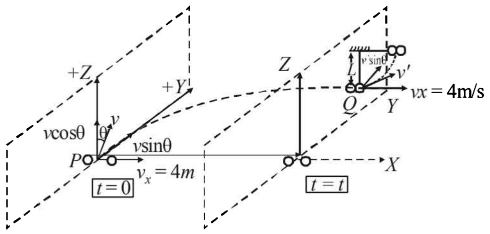
$u_y = 0, s_y = d, a_y = g, t_y = ?$

$S = ut + \dfrac{1}{2}at^2 \Rightarrow d = \dfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\dfrac{2d}{g}}$

The horizontal distance travelled by mass m during this time t

$x = 3\sqrt{6\mu gd} \times \sqrt{\dfrac{2d}{g}} = 6\sqrt{3\mu d^2} = 6d\sqrt{3\mu}$

7. When the stone reaches the point Q, the component of velocity in the $+Z$ direction ($v\cos\theta$) becomes zero due to the gravitational force in the $-Z$ direction.



The stone has two velocities at Q

• v_x in the $+X$ direction (4 m/s)

• $v\sin\theta$ in the $+Y$ direction ($6\sin 30° = 3$ m/s)

The resultant velocity of the stone

$v' = \sqrt{(v_x)^2 + (v\sin\theta)^2} = \sqrt{4^2 + 3^2} = 5\,\text{m/s}$

(i) Applying conservation of linear momentum at Q for collision with a mass of equal magnitude

$m \times 5 = 2m \times v$

Note : Since, the collision is completely inelastic the two masses will stick together. v is the velocity of the two masses just after collision.]

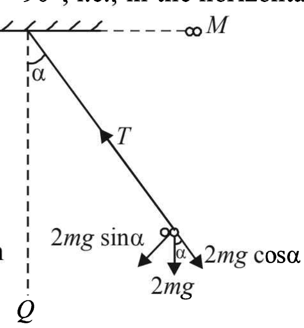
$\therefore \quad v = 2.5$ m/s

(ii) When the string is undergoing circular motion, at any arbitrary position $T - 2mg\cos\alpha = \dfrac{2mv^2}{\ell}$

Given that, $T = 0$ when $\alpha = 90°$ $\therefore 0 - 0 = \dfrac{2mv^2}{\ell} \Rightarrow v = 0$

$\Rightarrow$ Velocity is zero when α = 90°, i.e., in the horizontal position.

Applying energy conservation from Q to M, we get



$\dfrac{1}{2}2mv^2 = 2mg\ell$

$\Rightarrow \ell = \dfrac{v^2}{2g} = \dfrac{(2.5)^2}{2 \times 9.8} = 0.318\,\text{m}$

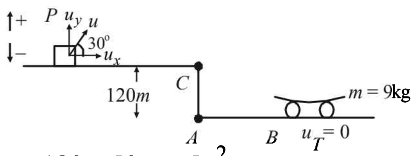
8. Consider the vertical motion of the cannon ball

$u_y = +100 \sin 30°$

$s_y = -120$ m

$a_y = -10$ m/s^2

$t = t_0$



$s = ut + \frac{1}{2}at^2 \quad \therefore -120 = 50\, t_0 - 5\, t_0^2$

$\Rightarrow \quad t_0^2 - 10\, t_0 - 24 = 0$

$\therefore \quad t_0 = -\frac{(-10) \pm \sqrt{100 - 4(1)(-24)}}{2} = 12 \text{ or } -2$ [Not valid]

$\therefore \quad t_0 = 12$ sec.

The horizontal velocity of the cannon ball remains the same

$\therefore \quad u_x = 100 \cos 30° = 50\sqrt{3}$ ms^{-1}

$\therefore$ Applying conservation of linear momentum to the cannon ball-trolley system in horizontal direction. If m is the mass of cannon ball and M is the mass of the trolley then,

$mu_x + M \times 0 = (m + M)\, v_x$

$\therefore \quad v_x = \frac{mu_x}{m + M}$ where v_x is the velocity of the cannon ball-trolley system.

$\therefore \quad v_x = \frac{1 \times 50\sqrt{3}}{1 + 9} = 5\sqrt{3}$ m/s

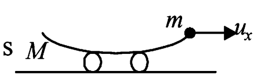
The second ball was projected after 12 second. Horizontal distance covered by the car P

$= 12 \times 5\sqrt{3} = 60\sqrt{3}$ m

Since the second ball also struck the trolley,
Therefore, in time 12 seconds, the trolley covers a distance of $60\sqrt{3}$ m.

For trolley after 12 seconds;

$u = 5\sqrt{3}$ m/s, $v = ?$, $t = 12$ s



$s = 60\sqrt{3}$ m, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 60\sqrt{3} = 5\sqrt{3} \times 12 + \frac{1}{2} \times a \times 144$

$\therefore \quad a = 0$ m/s^2 $\therefore v = u + at = 5\sqrt{3}$ m/s.

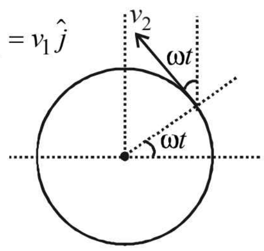
To find the final velocity of the carriage after the second impact we again apply conservation of linear momentum in the horizontal direction

$mu_x + (M + m)\, v_x = (M + 2m)\, v_f$

$\therefore \quad 1 \times 50\sqrt{3} + (9 + 1)\, 5\sqrt{3} = (9 + 2)\, v_f$

$\Rightarrow \quad v_f = 15.75$ m/s

9. $\vec{v}_2 = (-v_2 \sin \omega t\, \hat{i} + v_2 \cos \omega t\, \hat{j}); \vec{v}_1 = v_1 \hat{j}$



$\vec{v}_{PM} = \vec{v}_2 - \vec{v}_1$

$= -v_2 \sin \omega t\, \hat{i} + (v_2 \cos \omega t - v_1)\, \hat{j}$

$\vec{p}_{PM} = m\vec{v}_{PM}$

$= -mv_2 \sin \omega t\, \hat{i} + m(v_2 \cos \omega t - v_1)\, \hat{j}$

where $\omega = \frac{v_2}{R}$

or, $\vec{p}_{PM} = m\left[\left(-v_2 \sin \frac{v_2}{R} t\, \hat{i}\right) + \left(v_2 \cos \frac{v_2}{R} t - v_1\right) \hat{j}\right]$

H. Assertion & Reason Type Questions

1. **(d)** **Statement 1 :** For an elastic collision, the coefficient of restitution = 1

$e = \frac{|v_2 - v_1|}{|u_1 - u_2|} \Rightarrow |v_2 - v_1| = |u_1 - u_2|$

$\Rightarrow$ Relative velocity after collision is equal to relative velocity before collision. But in the statement relative speed is given.

Statement 2 : Linear momentum remains conserved in an elastic collision. This statement is true.

I. Integer Value Correct Type

1. **5** Velocity at the highest point of bob tied to string ℓ_1 is acquired by the bob tied to string ℓ_2 due to elastic head-on collision of equal masses

Therefore $\sqrt{g\ell_1} = \sqrt{5g\ell_2} \quad \therefore \frac{\ell_1}{\ell_2} = 5$

Section-B JEE Main/ AIEEE

1. **(d)** Let n be the number of bullets that the man can fire in one second.

$\therefore$ change in momentum per second $= n \times mv = F$

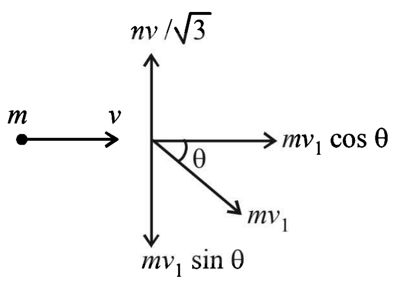
[m= mass of bullet, v = velocity] ($\because$ F is the force)

$$\therefore n = \frac{F}{mv} = \frac{144 \times 1000}{40 \times 1200} = 3$$

2. **(d)** In x-direction, $mv = mv_1 \cos\theta$ ...(1)

where v_1 is the velocity of second mass

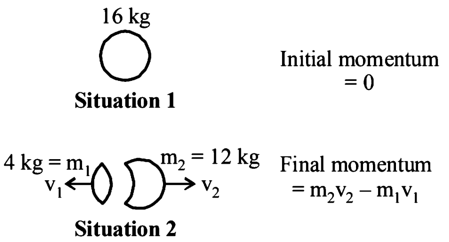
In y-direction, $\frac{mv}{\sqrt{3}} = mv_1 \sin\theta$ ...(2)



Squaring and adding eqns. (1) and (2)

$$v_1^2 = v^2 + \frac{v^2}{\sqrt{3}} \Rightarrow v_1 = \frac{2}{\sqrt{3}}v$$

3. **(b)** Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.



Applying conservation of linear momentum

$$m_2v_2 = m_1v_1 \Rightarrow v_1 = \frac{12 \times 4}{4} = 12\ ms^{-1}$$

$$\therefore K.E._1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2} \times 4 \times 144 = 288\ J$$

4. **(a)** In completely inelastic collision, all energy is not lost (so, statement -1 is true) and the principle of conservation of momentum holds good for all kinds of collisions (so, statement -2 is true) . Statement -2 explains statement -1 correctly because applying the principle of conservation of momentum, we can get the common velocity and hence the kinetic energy of the combined body.

5. **(b)** During each collision

$$\text{Initial velocity} = \frac{2}{2} = 1\, ms^{-1}$$

$$\text{Final velocity} = -\frac{2}{2} = -1\ ms^{-1}$$

Impulse = Change in momentum

$= m|v_2 - v_1| = 0.4 \times 2 = 0.8$ Ns

6. **(d)** Maximum energy loss $= \frac{P^2}{2m} - \frac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

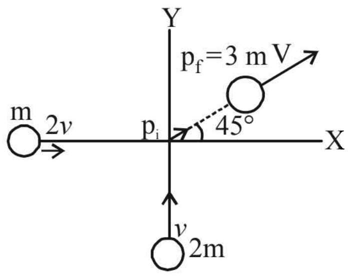
$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision. By comparing the equation given in statement I with above equation, we get

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

7. **(a)**



Initial momentum of the system

$$p_i = \sqrt{[m(2V)^2 \times m(2V)^2]}$$

$= \sqrt{2}m \times 2V$

Final momentum of the system = 3mV

By the law of conservation of momentum

$2\sqrt{2}mv = 3mV$

$$\Rightarrow \frac{2\sqrt{2}v}{3} = V_{combined}$$

Loss in energy

$$\Delta E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 - \frac{1}{2}(m_1 + m_2)V_{combined}^2$$

$$\Delta E = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$$

Percentage loss in energy during the collision = 56%